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How good are one-dimensional Josephson junction models?

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A two-dimensional model of Josephson junctions of overlap type is presented and shown to reduce to the usual one-dimensional (1D) model in the limit of a very narrow junction. Comparisons between the stability limits for fluxon reflection obtained from the two models suggest that the many results obtained from the one-dimensional model can be used for large-area junctions, thus explaining the remarkable agreement between 1D theory and experiments.

One of the most successful models in the area of nonlinear wave dynamics is the perturbed one-dimensional (1D) sine-Gordon equation modeling fluxon dynamics in Josephson junctions.[1,2] In particular, the overlap geometry junction has been analyzed in the framework of this model under the assumption that the width w of the junction is much smaller than the Josephson penetration depth λ_J. The results from this model agree very well with experimental findings despite the fact that the width of the junction often is comparable with λ_J. This is somewhat unexpected—and it is the purpose of this communication to clarify the range of validity for the 1D sine-Gordon–based Josephson junction model. We do this by introduction of a full two-dimensional model for the junctions of overlap type. From this model it follows that the stationary fluxon velocity and the threshold current for fluxon reflection (or existence of zero field steps) is almost independent of the width. This is due to the fact that the two-dimensional model reduces to one dimension when $\frac{1}{2}\eta(w/2)^2 \ll 1$, η being the uniform bias current through the junction—so we claim that the many results obtained under the tacit assumption are valid in a broader context. In order to support this statement we have investigated the stability of two-dimensional fluxons to reflection and compared the results with the results obtained from one dimension. The difference between one- and two-dimensional models is in agreement with our analysis.

The model is developed in Ref. 3 and is briefly described here. The geometry of an overlap junction is shown in Fig. 1. The tunneling supercurrent is described by the two basic Josephson equations:

$$j = j_J \sin\phi \tag{1a}$$

$$\frac{\partial\phi}{\partial t} = \frac{2e}{\hbar}V, \tag{1b}$$

where $\phi = \phi(x,y,t)$ is the difference between the phases of the order parameters of the two superconductors, $j = j(x,y,t)$ is the Josephson current crossing the insulating layer per unit area, j_J being the maximum supercurrent density. The voltage across the insulating barrier is given by $V = V(x,y,t)$; the constants e and $\hbar$ are the charge of the electron and Planck's constant divided by 2π, respectively. The surface current density $\mathbf{i} = \mathbf{i}(x,y,t)$ is given by

$$\mathbf{i} = \mathbf{H}\times\mathbf{n} = \frac{1}{\mu_0}(B_x, B_y)\times\mathbf{n} = \frac{\hbar}{2ed\mu_0}\nabla\phi. \tag{2}$$

Here $d = 2\lambda_L + t_0$ is the magnetic thickness of the junction, λ_L is the London penetration depth, μ_0 is the permeability of free space, and B_x (B_y) is the x component (y component) of $\mathbf{B}$. The unit vector normal to the surface is denoted $\mathbf{n}$.

The current density through the oxide layer is given by

$$j_z = j_J \sin\phi + \frac{\hbar}{2eR}\phi_t + \frac{\hbar C}{2e}\phi_{tt}; \tag{3}$$

the second term on the right-hand side represents dissipative effects due to quasi-particle tunneling, R being an effective normal resistance. The third term represents the energy stored in the barrier (i.e., a displacement current).

From now on all the lengths x, y, t_0, λ_L, t_f, ω, L are measured in units of the Josephson penetration depth $\lambda_J = (\hbar/2\mu_0 e d j_J)^{1/2}$ and time t in units of the reciprocal plasma frequency ω_p^{-1}, where $\omega_p = (2ej_J/\hbar C)^{1/2}$. Finally, the equation of continuity

$$\nabla\cdot\mathbf{i} - j = 0 \tag{4}$$

yields the two-dimensional sine-Gordon equation

$$\phi_{xx} + \phi_{yy} = \sin\phi + \phi_{tt} + \alpha\phi_t, \tag{5}$$

where $\alpha = (\hbar/2eR)\omega_p = 1/\sqrt{\beta}_C$, and β_C is the usual McCumber parameter.

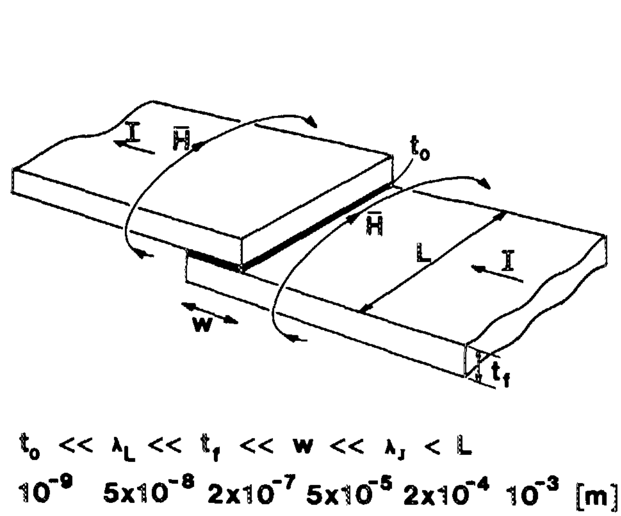

FIG. 1. Josephson junction of overlap type.

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The energy input to the system is provided from the magnetic fields induced at the boundaries which are modelled by the boundary conditions

$$H\left(x,\pm\frac{w}{2},t\right)\cong\pm\eta\frac{w}{2}=\pm\phi_y\left(x,\frac{w}{2},t\right)=\pm\beta \qquad (6)$$

for $|x|\leqslant L/2$ and $\eta = I/Lwj_J$ (I being the total bias current fed in the y direction, L and w are length and width of the junction) is the uniform bias current through the junction. From Ampere's law it follows that

$$H\left(\pm\frac{L}{2},y,t\right)=\phi_x\left(\pm\frac{L}{2},y,t\right)\cong 0. \qquad (7)$$

Thus for the two-dimensional model the energy providing mechanism is the magnetic field via the boundary conditions (6). Now, it is easily shown that Eq. (5) reduces to the one-dimensional sine-Gordon equation if the ansatz $\phi(x,y,t)=\phi_1(x,t)+\eta y^2/2$ is used and

$$\delta\equiv\frac{\eta}{2}\left(\frac{w}{2}\right)^2\ll 1. \qquad (8)$$

The restriction (8) is weaker than the usual assumption $\delta\equiv w\ll 1$. Further it can be shown that the magnitude of the critical current obtained from the two-dimensional model in the limit $w\to 0$ is $\eta_c = 1$ (in accordance with the one-dimensional model) and for $w\gg 1$ is $\eta_c = 4/w$.[3,4] If η exceeds the critical current fluxons and antifluxons will be generated at the boundaries $y = w/2$ and $y = (-w/2)$ and the phase as a whole will start to increase. Thus one stability criterion for fluxon propagation in a two-dimensional Josephson junction is that

$$\beta<\frac{\eta_c w}{2}. \qquad (9)$$

The very important question in connection with Josephson junctions is the stability of fluxon propagation when the junction is biased on a zero field step. In this case Eq. (9) will be fulfilled but there will be a lower limit for β corresponding to the situation where the fluxon annihilates at the boundary $x=\pm L/2$. In the following we determine this lower limit for β using the two-dimensional model by means of numerical calculations. We remark that in Ref. 3 it is shown that the two-dimensional fluxon (due to energy dissipation) is forced to have orientation perpendicular to the long direction of the junction (the x axis) and that the stationary velocity at the two-dimensional fluxon only differs a few percent from the velocity obtained from the one-dimensional model. In the numerical solution we have applied an explicit formula based on a stabilized leap-frog scheme extended to two spatial steps $\sqrt{2}/32$. In Fig. 2 we show a two-dimensional fluxon propagating in the negative x direction. The parameter values are $\alpha = 0.1, \beta = 0.6, L = 40, w = 12$, and the initial velocity of the fluxon is obtained from the one-dimensional

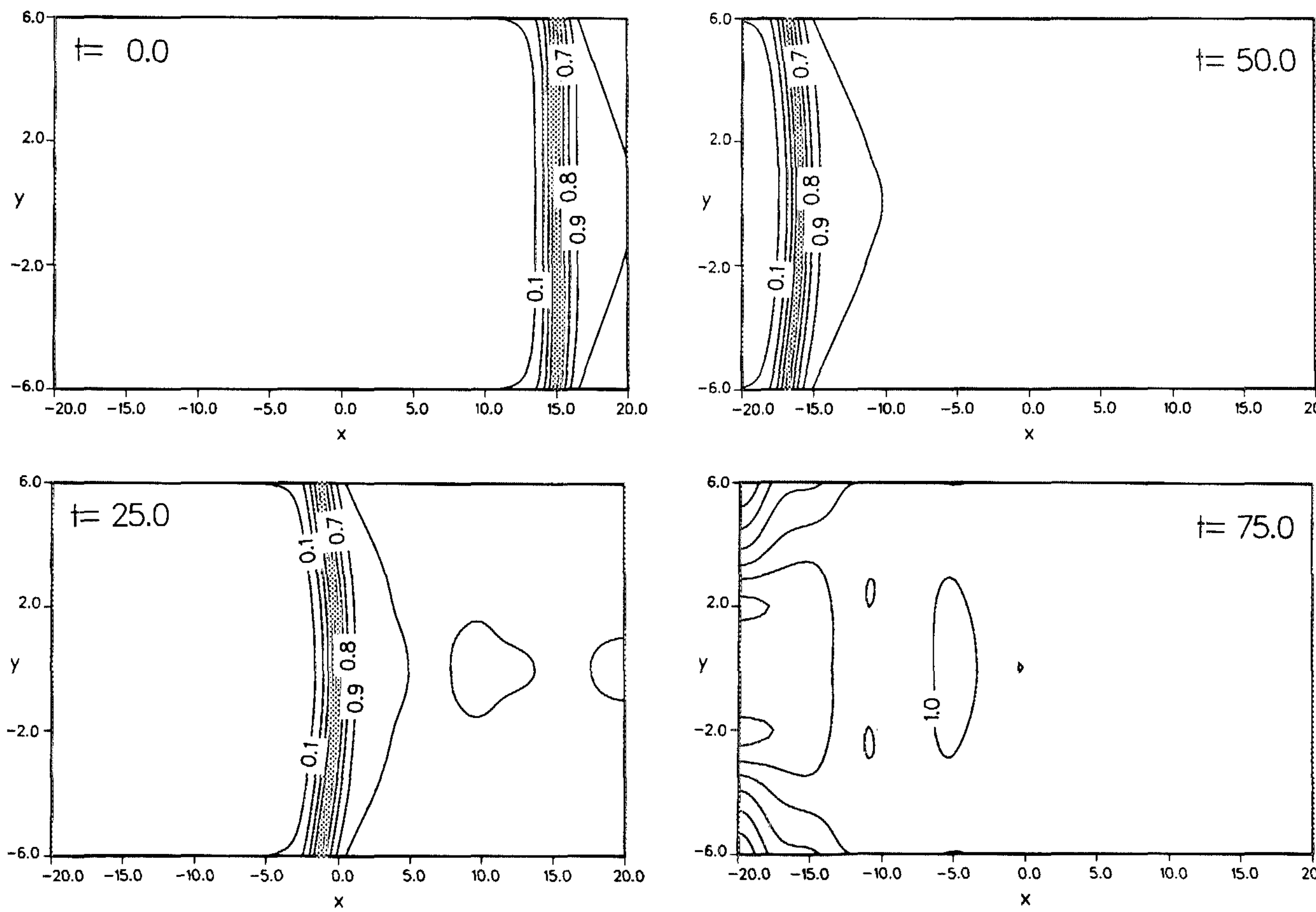

FIG. 2. Two-dimensional fluxon propagating in a rectangular Josephson junction. Parameter values $w = 12$, $L = 40$, $\alpha = 0.1$, $\beta = 0.6$. Fluxon initially placed at $x = 15$ and started with the one-dimensional power balance velocity. Bias current is in the direction of the y axis. Results are displayed in terms at contour plots where $\phi/2\pi$ equals multiples at 0.1 along the curves. Snapshots are shown for $t = 0, 25, 50$, and 75. After transients the fluxon is u shaped. The fluxon annihilates at the boundary.

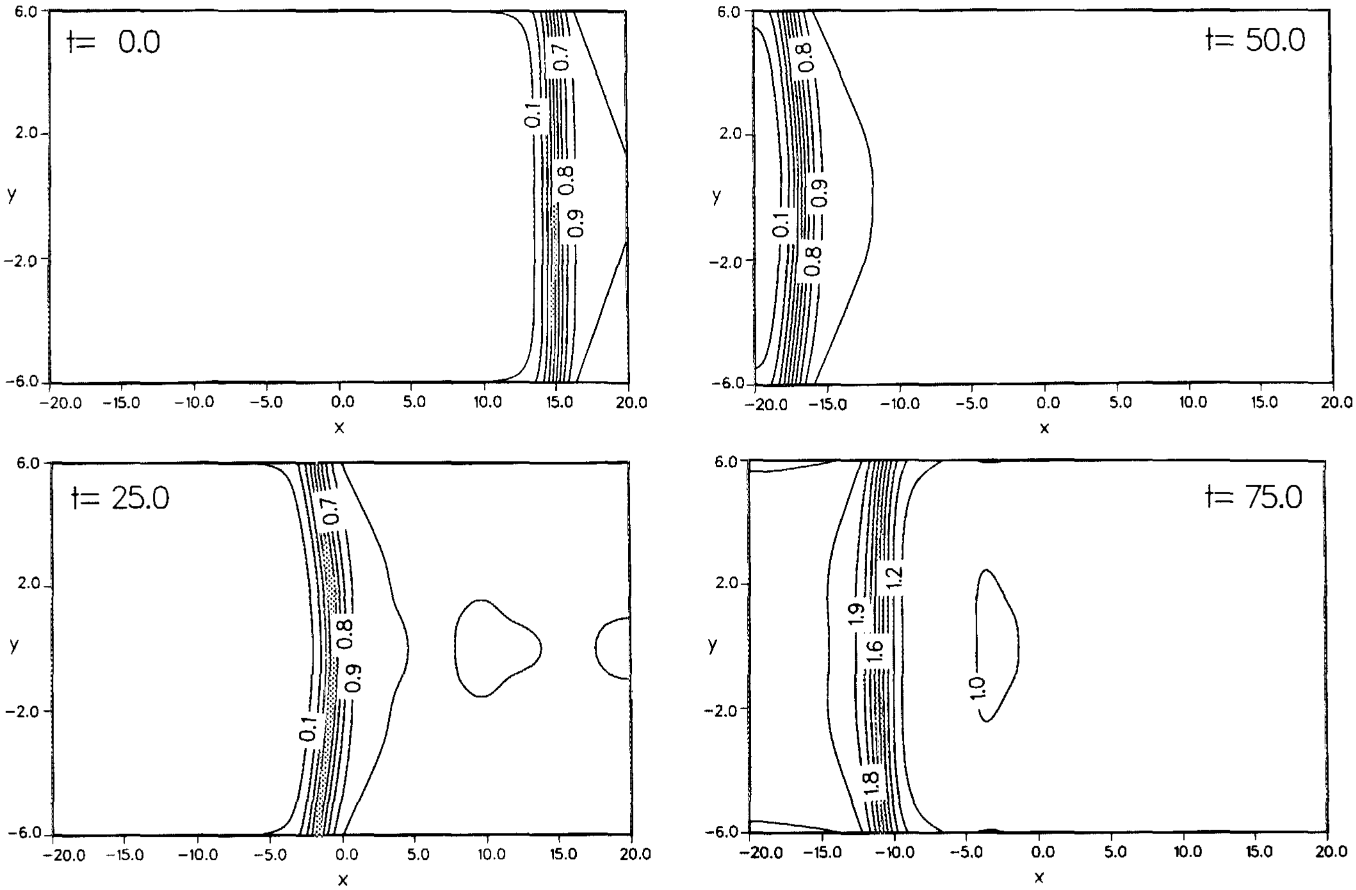

FIG. 3. Parameter values as in Fig. 2 except for $\beta = 0.62$. The fluxon is reflected due to the higher energy input.

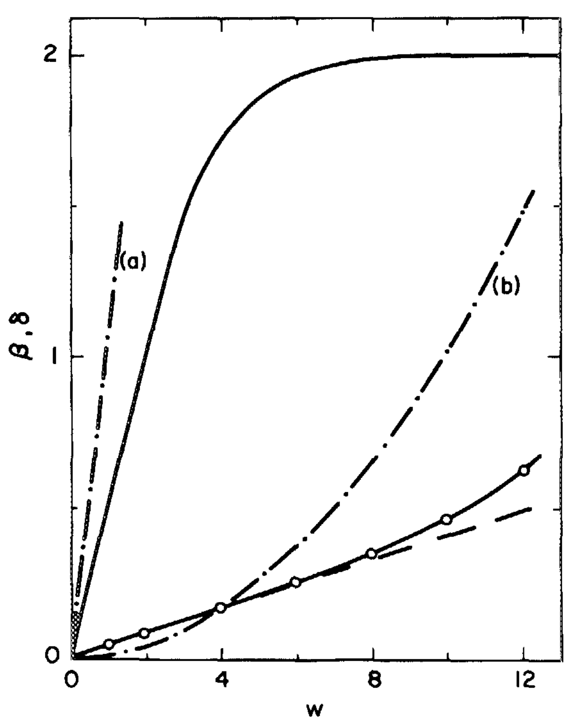

FIG. 4. Stability limits for a two-dimensional fluxon. In the region between the full curves (β vs w) the first zero field step is expected: the upper limit corresponds to a switch to the ohmic line, the lower limit (numerically determined) to fluxon annihilation. The dashed line is the limit obtained from the one-dimensional model $\beta = \eta_{1D, C}\,(w/2)$, $\eta_{1D, C} = 0.082$ (1). Dash-dotted lines: (a) $\delta = w$ (the usual assumption for the validity of the 1D model), (b) $\delta = \frac{1}{2}\eta_{1DC}(w/2)^2$ the criterion derived in this investigation. The difference between 1D and 2D models is negligible for $\delta = \frac{1}{2}\eta_{1D,C}(w/2)^2 \ll 1$.

power-balance velocity $(u_\infty = \{1 + [(4/\pi)(\alpha/\eta]^2\}^{-1/2} = \{1 + [(2/\pi)(\alpha w/\beta)]^2\}^{-1/2})$. The fluxon is seen to annihilate at the boundary. In Fig. 3 the fluxon survives the reflection—the parameter values are as in Fig. 2 except for $\beta = 0.62$. Figure 4 shows the stability limits and a comparison between results obtained from one- and two-dimensional models. The area between the full curves is our estimate of the region for existence of zero field steps. The upper curve is taken from Ref. 4; the lower curve is numerically determined. The dotted curve is the limit for fluxon reflection obtained from the one-dimensional model. Dash-dotted lines represent the usual restriction and the restriction from the present analysis for the validity at the one-dimensional model. The agreement between the models is excellent when Eq. (8) is fulfilled supporting the statement above.

The analysis in this paper indicates that the results from the one-dimensional model developed under the assumption $w \ll 1 (=\lambda_J)$ are valid in a much larger range, namely for junctions where Eq. (8) is fulfilled. This is supported by experiments which seem insensitive to the width of the junction.

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